Polynomial Estimation of Time-Varying Multipath Gains With Intercarrier Interference Mitigation in OFDM Systems

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Abstract—In this paper, we consider the case of a high-speed mobile receiver operating in an orthogonal frequency-division multiplexing (OFDM) communication system. We present an iterative algorithm for estimating multipath complex gains with intersubcarrier interference (ICI) mitigation (using comb-type pilots). Each complex gain variation is approximated by a polynomial representation within several OFDM symbols. Assuming knowledge of delay-related information, polynomial coefficients are obtained from time-averaged gain values, which are estimated using the least-square (LS) criterion. The channel matrix is easily computed, and the ICI is reduced by using successive interference suppression (SIS) during data symbol detection. The algorithm's performance is further enhanced by an iterative procedure, performing channel estimation and ICI mitigation at each iteration. Theoretical analysis and simulation results for a Rayleigh fading channel show that the proposed algorithm has low computational complexity and good performance in the presence of high normalized Doppler spread.

Index Terms—Channel estimation, intersubcarrier interference (ICI), orthogonal frequency-division multiplexing (OFDM), successive interference suppression (SIS), time-varying channels.

I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) is an attractive technique for high-speed data transmission in mobile communications. Currently, OFDM has been adapted to digital audio and video broadcasting (DAB/DVB) systems, to high-speed wireless local area networks (WLANs) such as IEEE 802.11x and HIPERLAN/2, and to multimedia mobile access communication (MMAC), asymmetric digital subscriber lines (ADSLs), digital multimedia broadcasting (DMB), multiband OFDM-type ultrawideband (MB-OFDM UWB) systems, etc. In OFDM systems, each subcarrier has a narrow bandwidth, which makes the signal robust against frequency selectivity, which can arise from multipath delay spread. However, OFDM is relatively sensitive to time-domain selectivity, which is induced by rapid temporal variations of a mobile channel. Such variations corrupt the orthogonality of the OFDM subcarrier waveforms, leading to intersubcarrier interference (ICI).

In the case of wideband mobile communication systems, dynamic channel estimation is needed because the radio channel is frequency selective and time varying [5]. In practice, the channel may significantly change, even within one OFDM symbol. It is thus preferable to estimate the channel by inserting pilot tones, called comb-type pilots, into each OFDM symbol [6]. Assuming such a strategy, conventional methods generally consist of estimating the channel at pilot frequencies and then interpolating [8] the channel frequency response. The estimation of the channel at pilot frequencies can be based on least square (LS) or linear minimum mean square error (LMMSE). LMMSE has been shown to have better performance than LS [6]. In [7], the complexity of LMMSE is reduced by deriving an optimal low-rank estimator with singular value decomposition.

In [9], the channel estimator is based on a parametric channel model, which directly estimates the time delays and complex attenuations of the multipath channel. This estimator yields the best performance from all comb-type pilot channel estimators as long as the channel remains invariant within one OFDM symbol.

Recently, the basis expansion model (BEM) was introduced to approximate OFDM channel variations. First, for slow fading assumptions, Wang and Liu [16] used a polynomial basis function model for the channel response in a time–frequency window, whereas Senol et al. [17] modeled the correlated discrete-time fading channel using a Karhunen–Loeve (KL) orthogonal expansion.

For fast time-varying channels, many existing works resort to estimating the equivalent discrete-time channel taps, which are modeled by the BEM [18], [19]. The BEM methods [18] are KL BEM (KL-BEM), prolate spheroidal BEM (PS-BEM), complex-exponential BEM (CE-BEM), and polynomial BEM (P-BEM). KL-BEM is optimal in terms of mean square error (MSE) but is not robust to statistical channel mismatches, whereas PS-BEM is a general approximation for all kinds of channel statistics, although its band-limited orthogonal spheroidal functions have maximal time concentration within the considered interval. CE-BEM is independent of channel statistics but induces a large modeling error. Finally, a great deal of attention has been paid to P-BEM [19], although its
modeling performance is rather sensitive to the Doppler spread; nevertheless, it provides a better fit for low than for high Doppler spreads. In [22], a piecewise linear method is used to approximate the channel taps, and the channel tap slopes are estimated from the cyclic prefix (CP) or from adjacent OFDM symbols.

For ICI mitigation, MMSE and successive interference cancellation (SIC) schemes with optimal ordering were developed in [23]. Since the number of subcarriers is usually very large, this receiver is highly complex. In [24] and [25], a low-complexity MMSE and decision-feedback equalizer (DFE) were developed based on the fact that most of a symbol’s energy is distributed over just a few subcarriers, and that the ICI on a subcarrier mainly originates from its neighboring subcarriers. These equalizers are in the case of pure Doppler-induced ICI (i.e., with sufficient guard interval). In the case of insufficient CP, intersymbol interference (ISI) occurs and can lead to a considerable performance degradation. In [26], the authors suggest an iterative technique for the equalization of ICI and ISI.

As the channel delay spread increases, the number of channel taps also increases, thus leading to a large number of BEM coefficients [18]. In such a case, more pilot symbols are needed to estimate the BEM coefficients. In contrast to the research described in [18], we sought to directly estimate the physical channel instead of the equivalent discrete-time channel taps. This means estimating the physical propagation parameters such as multipath delays and multipath complex gains. For a fast time-varying channel, the channel matrix in the OFDM system depends on multipath delays and time variations of the multipath complex gains within a single OFDM symbol. In [2], we proposed an algorithm for channel matrix estimation and ICI reduction, which is executed per block of OFDM symbols. Assuming the availability of delay information, the time-varying complex gains within a given OFDM symbol are obtained by interpolating the estimated time-averaged values over each symbol of the block. This algorithm is very demanding in terms of computing power.

In this paper, we present a new low-complexity iterative algorithm for the estimation of complex gains with ICI mitigation in OFDM downlink mobile communication systems that use comb-type pilots. By exploiting the nature of the channel, the delays are assumed to be invariant and perfectly estimated, as we have done in OFDM [2] and code-division multiple-access (CDMA) [3], [4] contexts. It should be noted that an initial and generally accurate estimation of the number of paths and time delays can be obtained by using the minimum description length (MDL) and estimation of signal parameters by rotational invariance techniques (ESPRIT) methods [9], [11]. First, we compute the time average of the complex gains over the effective duration of the OFDM symbol by using the LS criterion as was done in [2]. Then, we show that the time variation of each complex gain can be approximated in a polynomial fashion within several OFDM symbols, where the coefficients of each polynomial are calculated from the estimated time-averaged values. Hence, thanks to the use of polynomial modeling, the channel matrix can be computed with low complexity from the estimated coefficients, and the ICI is reduced using successive interference suppression (SIS) in data symbol detection. We provide theoretical and simulated MSE multipath channel complex gain estimation analysis expressed in terms of the normalized (with respect to the OFDM symbol time) Doppler spread. By taking advantage of an iterative procedure, at each step of which the ICI is estimated and then removed, the algorithm proposed here has demonstrated considerable improvements in performance while reducing the computational complexity when compared with that described in [2].

The organization of this paper is as follows. Section II introduces the OFDM baseband model, whereas Section III describes the polynomial modeling. Section IV covers the algorithm used to estimate the polynomial coefficients as well as the iterative algorithm. Section V presents the results of simulations that validate our technique. Finally, our conclusions are presented in Section VI.

**Notation:** The notations used in this paper are as follows. Upper (lower) boldface letters denote matrices (column vectors). $X_k$ denotes the $k$th element of the vector $x$, and $X_{k,m}$ denotes the $(k,m)$th element of the matrix $X$. $I_N$ is a $N \times N$ identity matrix, and $\text{diag}(x)$ is a diagonal matrix with $x$ on its main diagonal. The superscripts $(\cdot)^T$ and $(\cdot)^H$ stand, respectively, for transpose and Hermitian operators. $|\cdot|$, $\text{Tr}(\cdot)$, and $\mathbb{E}[\cdot]$ are the determinant, trace, and expectation operators, respectively, and $\text{Re}(\cdot)$, $\| \cdot \|$, and $(\cdot)^*$ are the real part, magnitude, and conjugate of a complex number or matrix, respectively. $\|X\|^2_F$ is the Frobenius matrix norm, $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, and $\delta_{k,m}$ is the Kronecker symbol.

**II. System Model**

If we consider an OFDM system with $N$ subcarriers, the duration of an OFDM symbol can be written as $T = vT_s$, with $v = N + N_g$, where $N_g$ is the length of the CP, and $T_s$ is the sampling time. On the transmitter side, an $N$-point inverse fast Fourier transform (IFFT) is applied to a normalized quadratic-amplitude modulation (QAM) symbol data block $\{x_n[b]\}$ (i.e., $\mathbb{E}[x_n[b]x_n[b]^{*}] = 1$), where $n$ and $b$ represent, respectively, the OFDM symbol index and the subcarrier index. A CP, which is a copy of the last samples of the IFFT output, is added to avoid the ISI caused by multipath fading channels. The output baseband signal of the transmitter can be represented as

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{q=-N_g}^{N-1} s(n)[q]g_e(t - qT_s - nT) \quad (1)$$

where $g_e(t)$ is the impulse response of the transmission analog filter, and $s(n)[q]$ with $q \in [-N_g, N-1]$ are the $(N + N_g)$ samples of the IFFT output completed by the CP of the $n$th OFDM symbol, which is given by

$$s(n)[q] = \frac{1}{N} \sum_{b=-\frac{N_g}{2}}^{\frac{N_g}{2}} x(n)[b]e^{j2\pi bq/N}. \quad (2)$$
It is assumed that the signal is transmitted over a multipath Rayleigh fading channel characterized by

$$h(t, \tau) = \sum_{l=1}^{L} \alpha_l(t) \delta(\tau - \tau_l T_s)$$  \hspace{1cm} (3)$$

where $L$ is the total number of propagation paths, $\alpha_l$ is the $l$th complex gain of variance $\sigma^2_{nl}$, and $\tau_l$ is the $l$th delay normalized by the sampling time ($\tau_l$ is not necessarily an integer). The $L$ individual elements of $\{\alpha_l(t)\}$ are uncorrelated with respect to each other. They are wide-sense stationary (WSS) narrowband complex Gaussian processes with the so-called Jakes’ power spectrum of maximum Doppler frequency $f_d$ [10]. The average energy of the channel is normalized to 1 (i.e., $\sum_{l=1}^{L} \sigma^2_{nl} = 1$).

On the receiver side, after passing to discrete time by means of low-pass filtering and A/D conversion, the CP is removed assuming that its length is no less than the maximum delay. Afterward, an $N$-point fast Fourier transform (FFT) is applied to transform the time sequence into the frequency domain. If we consider that the $N$ transmission subcarriers are within the flat region of the frequency response of each of the transmitter and receiver filters, then, omitting the time index $n$, the $N$ received subcarriers are given by [2], [9]

$$y(t) = Hx + w(t)$$  \hspace{1cm} (4)$$

where $x, y,$ and $w$ are $N \times 1$ vectors given by

$$x = \begin{bmatrix} x \left[ -\frac{N}{2} \right], x \left[ -\frac{N}{2} + 1 \right], \ldots, x \left[ \frac{N}{2} - 1 \right] \end{bmatrix}^T$$

$$y = \begin{bmatrix} y \left[ -\frac{N}{2} \right], y \left[ -\frac{N}{2} + 1 \right], \ldots, y \left[ \frac{N}{2} - 1 \right] \end{bmatrix}^T$$

$$w = \begin{bmatrix} w \left[ -\frac{N}{2} \right], w \left[ -\frac{N}{2} + 1 \right], \ldots, w \left[ \frac{N}{2} - 1 \right] \end{bmatrix}^T$$

and $H$ is an $N \times N$ matrix with elements given by

$$[H]_{k,m} = \frac{1}{N} \sum_{l=1}^{L} \left[ e^{-j2\pi \left( \frac{m-1}{N} + \frac{k-1}{N} \right)} \right]^{N-1} \sum_{q=0}^{N-1} \alpha_l(q T_s) e^{j2\pi \frac{m-1}{N} q}$$  \hspace{1cm} (5)$$

where $\{\alpha_l(q T_s)\}$ is the $T_s$ spaced sampling of the $l$th complex gain value, and $w[b]$ is the white complex Gaussian noise with variance $\sigma^2$. The channel matrix contains the time average of the channel frequency response $[H]_{k,m}$ on its diagonal and the coefficients of CI $[H]_{k,m}$ for $k \neq m$. It should be noted that $H$ would clearly be a diagonal matrix if the complex gains were time invariant within one OFDM symbol.

III. COMPLEX GAIN POLYNOMIAL MODELING

In this section, we show that, for realistically high Doppler spread $f_d T_s$, each sampled complex gain $\alpha_l = \left[ \alpha_l(-N_g T_s), \ldots, \alpha_l((vN_c - N_g - 1) T_s) \right]^T$ within $N_c$ OFDM symbols can be approximated by a polynomial model containing $N_c$ coefficients (i.e., a $(N_c - 1)$ degree polynomial). Thus, for $q \in \mathbb{D} = [-N_g, vN_c - N_g - 1]$, $\alpha_l(q T_s)$ can be expressed as

$$\alpha_l(q T_s) = \sum_{d=0}^{N_c-1} c_{d,l} q^d + \xi_l[q]$$  \hspace{1cm} (6)$$

where $c_l = [c_{0,l}, \ldots, c_{N_c-1,l}]^T$ are the $N_c$ polynomial coefficients, and $\xi_l[q]$ is the model error. We will also show that a good approximation can be obtained by calculating the $N_c$ coefficients from only $\overline{\alpha}_l = [\overline{\alpha}_{1,l}, \ldots, \overline{\alpha}_{N_c-1,l}]^T$, where $\overline{\alpha}_{l,d} = (1/N) \sum_{q=d}^{N-1} \alpha_l(q T_s)$ is the time average computed over the effective duration of the $(d + 1)$th OFDM symbol of the $l$th complex gain.

A. Optimal Polynomial

The optimal polynomial $\alpha_{opt}$, which is LS fitted (linear and polynomial regression) [15] to $\alpha_l$, and its $N_c$ coefficients $c_{opt,l}$ are given by

$$\alpha_{opt,l}(q) = Q^T c_{opt,l} = S \alpha_l$$

$$c_{opt,l} = (Q^T Q)^{-1} Q \alpha_l$$  \hspace{1cm} (7)$$

where $Q$ is an $N_c \times v N_c$ matrix of elements $[Q]_{k,m} = (m - (N_c - 1)(k - 1))$, and $S = Q^T(Q Q^T)^{-1} Q$ is a $v N_c \times v N_c$ matrix. It provides the MMSE approximation for all polynomials containing $N_c$ coefficients given by

$$\text{MMSE}_l = \frac{1}{v N_c} E \left[ (\alpha_l - \alpha_{opt,l})^H (\alpha_l - \alpha_{opt,l}) \right]$$

$$= \frac{1}{v N_c} \text{Tr} \left( (I_{v N_c} - S) R_{\alpha_l} (I_{v N_c} - S)^T \right)$$  \hspace{1cm} (8)$$

where $R_{\alpha_l} = E[\alpha_l \alpha_l^H]$ is the $v N_c \times v N_c$ correlation matrix of $\alpha_l$. Since $\alpha_l(t)$ is a WSS narrowband complex Gaussian process with the so-called Jakes’ power spectrum [10], then

$$[R_{\alpha_l}]_{k,m} = \sigma^2_{\alpha_l} J_0 \left( 2\pi f_d T_s (k - m) \right).$$  \hspace{1cm} (9)$$

B. Desired Polynomial

Our aim now is to find the polynomial approximation of $N_c$ coefficients solely based on the knowledge of $\overline{\alpha}_l$. This polynomial $\alpha_{des,l}$ and its coefficients $c_{des,l}$ are given by

$$\alpha_{des,l}(q) = Q^T c_{des,l} = V \overline{\alpha}_l$$

$$c_{des,l} = T^{-1} \overline{\alpha}_l$$  \hspace{1cm} (10)$$

where $T$ is the $N_c \times N_c$ transfer matrix between $c_{des,l}$ and $\overline{\alpha}_l$, and $V = Q^T T^{-1}$. For $N_c = 3$, $T$ is given by

$$T = \begin{bmatrix}
1 & 0 & 0 \\
\frac{N_c - 1}{2} + v & \frac{(N-1)(2N-1)}{6} + (N-1)v + v^2 & \frac{(N-1)(2N-1)}{6} + (N-1)v + 4v^2 \\
\frac{N_c - 1}{2} + 2v & \frac{(N-1)(2N-1)}{6} + 2(N-1)v + 4v^2 & \frac{(N-1)(2N-1)}{6} + 2(N-1)v + 9v^2
\end{bmatrix}.$$
modeling is given by Fig. 1. Comparison between MMSE and MSE

As shown in Fig. 1, even with just $L$ paths.

$\mathbf{T}$ matrix (defined for $N_c = 3$). The MSE of this polynomial modeling is given by

$$
\text{MSE}_{\text{des}} = \frac{1}{1/N_c} \mathbb{E} \left[ e_{\text{des}} e_{\text{des}}^H \right]
$$

$$
\approx \frac{1}{1/N_c} \text{Tr} \left( \mathbf{R}_\alpha + \mathbf{V} \mathbf{R}_\alpha \mathbf{V}^T - \mathbf{R}_{\alpha/\bar{\alpha}} \mathbf{V}^T - \mathbf{V} \mathbf{R}_{\alpha/\bar{\alpha}} \right) \tag{11}
$$

where $e_{\text{des}} = \alpha_t - \alpha_{\text{des}t}$ is the model error, $\mathbf{R}_\alpha$ is the $N_c \times N_c$ correlation matrix of $\alpha_t$, and $\mathbf{R}_{\alpha/\bar{\alpha}}$ is the $N_c \times N_c$ cross-correlation matrix between $\alpha_t$ and $\bar{\alpha}_t$ with elements given by

$$
[\mathbf{R}_\alpha]_{k,m} = \frac{\sigma_{\alpha}^2}{N} \sum_{q=1}^{k-1} \sum_{v=q}^{N-1} J_0 (2\pi f_d T_s (q_1 - q_2))
$$

$$
[\mathbf{R}_{\alpha/\bar{\alpha}}]_{k,m} = \frac{\sigma_{\alpha}^2}{N} \sum_{q=1}^{k-1} \sum_{v=q}^{N-1} J_0 (2\pi f_d T_s (k - q - N_g - 1)). \tag{12}
$$

As shown in Fig. 1, even with just $N_c = 2$ coefficients, we have $\text{MSE}_{\text{des}} \approx \text{MMSE}$, and for $f_d T \leq 0.1$, $\text{MSE}_{\text{des}} \leq 10^{-4}$. This proves that, for high realistic values of $f_d T$, we can approximate $\alpha_t$ by a polynomial model with $N_c$ coefficients and can calculate the polynomial approximation using only the time average values $\bar{\alpha}_t$. More explanation about polynomial modeling for Jakes’ process can be found in [1].

Under this polynomial approximation, the channel matrix [see (5)] for the $n$th $N_c$ OFDM symbols can simply be defined as

$$
\mathbf{H}_n = \frac{1}{N} \sum_{d=0}^{N_c-1} \mathbf{B}_{n,d} \tag{13}
$$

with

$$
\mathbf{B}_{n,d} = \mathbf{M}_{n,d} \mathbf{\text{diag}} \{\mathbf{F} \chi_d\}
$$

where $\chi_d = [c_{d,1}, \ldots, c_{d,L}]^T$, $\mathbf{F}$ is the $N \times L$ Fourier matrix, and $\mathbf{M}_{n,d}$ is an $N \times N$ matrix given by

$$
[\mathbf{F}]_{k,m} = e^{-j2\pi \left( \frac{k-1}{N} - \frac{m-1}{L} \right)}
$$

$$
[\mathbf{M}_{n,d}]_{k,m} = \sum_{q=0}^{N-1} (q + (n-1)v)^d e^{j2\pi \frac{m-1}{L} q} \tag{14}
$$

where $n \in [1, N_c]$. Notice that the terms of the matrix $\mathbf{M}_{n,d}$ can easily be computed and stored using the properties of power series. This simplified representation of the channel matrix will be used throughout the algorithm as we present in the next section.

IV. ESTIMATION OF POLYNOMIAL COEFFICIENTS AND THE ITERATIVE ALGORITHM

In this section, we propose a method based on comb-type pilots and multipath time delay information. This method consists of estimating the $N_c$ coefficients of the polynomial fitted to the time-averaged complex gains over the effective duration of $N_c$ OFDM symbols.

A. Pilot Pattern and Received Pilot Subcarriers

The $N_p$ pilot subcarriers are fixed during transmission and evenly inserted into $N$ subcarriers. As opposed to the methods described in [8] and [9], the distance $L_f$ (in frequency domain) between two adjacent pilots can be selected without the need to respect the sampling theorem. However, as shown in (20), $N_p$ must fulfill the following requirement: $N_p \geq L$.

Let $\mathcal{P}$ denote the set containing the index positions of the $N_p$ pilot subcarriers defined by

$$
\mathcal{P} = \{ p_s \} \mathcal{P}_s = (s-1)L_f + 1, \quad s = 1, \ldots, N_p \}. \tag{15}
$$

The received pilot subcarriers can be written as the sum of three components

$$
\mathbf{y}_p = \text{diag} \{ x_p \} \mathbf{h}_p + \mathbf{H}_p \mathbf{x} + \mathbf{w}_p \tag{16}
$$

where the $N_p \times 1$ vectors $x_p$, $\mathbf{y}_p$, and $\mathbf{w}_p$ are given by

$$
x_p = [x[p_1], x[p_2], \ldots, x[p_{N_p}]]^T
$$

$$\mathbf{y}_p = [y[p_1], y[p_2], \ldots, y[p_{N_p}]]^T
$$

$$\mathbf{w}_p = [w[p_1], w[p_2], \ldots, w[p_{N_p}]]^T.
$$

In the preceding equation, $\mathbf{h}_p$ is an $N_p \times 1$ vector and $\mathbf{H}_p$ is an $N_p \times N$ matrix with elements given by

$$
[\mathbf{h}_p]_k = [\mathbf{H}_{p_k}]_k, \quad \text{if } m \neq p_k
$$

$$
[\mathbf{H}_p]_{k,m} = \begin{cases} 
\mathbf{H}_{p_k,m}, & \text{if } m \neq p_k \\
0, & \text{if } m = p_k. \tag{17}
\end{cases}
$$

The first component is the desired term without ICI, and the second component is the ICI term. $\mathbf{h}_p$ can be written as the Fourier transform for different complex gain time averages $\alpha = [\bar{\alpha}_1, \ldots, \bar{\alpha}_L]^T$, i.e.,

$$
\mathbf{h}_p = \mathbf{F}_p \alpha \tag{18}
$$
where \( \alpha_l = \left( \frac{1}{N} \right)^{l} \sum_{q=0}^{N-1} \alpha_l (qT_s) \), and \( F_p \) is the \( N_p \times L \) Fourier transform matrix given by

\[
\begin{bmatrix}
F_p & \vdots & \vdots \\
\vdots & \ddots & \vdots \\
\vdots & \vdots & F_p
\end{bmatrix}
\]

(19)

\[ a_{\text{LS}} = G y_p \]

\[
G = (F_p^H \text{diag}(x_p) F_p) \left( F_p^H \text{diag}(x_p) F_p \right)^{-1} (F_p^H \text{diag}(x_p) F_p)^H
\]

(20)

where \( G \) is an \( L \times N_p \) matrix. By estimating \( a \) for \( N_c \) consecutive OFDM symbols, the \( N_c \) polynomial coefficients of each complex gain are obtained (as shown in Section III) by

\[
\hat{C}_{\text{des}} = T^{-1} A_{\text{LS}}
\]

(21)

where \( \hat{C}_{\text{des}} = [\hat{c}_{\text{des}1}, \ldots, \hat{c}_{\text{des}L}] \) and \( A_{\text{LS}} = [\alpha_{LS1}, \ldots, \alpha_{LSL}] \) are \( N_c \times L \) matrices.

C. Iterative Algorithm

In the iterative algorithm for channel estimation and ICI suppression, the OFDM symbols are grouped into blocks of \( N_c \) OFDM symbols each. The iterative algorithm is shown in Fig. 2, where \( \{r_{(n)}[q]\} \) is the received sampled signal without CP. The complete algorithm is divided into two modes, i.e., channel matrix estimation mode and detection mode, as shown in Fig. 2(a). The first of these involves the estimation of the \( N_c \) polynomial coefficients \( C_{\text{des}} \) by means of an LS estimator and computation of the channel matrix, as shown in Fig. 2(b).

The second mode involves the detection of data symbols using a successive data interference suppression (SIS) scheme with one-tap frequency equalizer (see Appendix C). A feedback technique is used between these two modes, iteratively performing ICI suppression and channel matrix estimation. The algorithm is executed in two stages, i.e., an initialization stage and a sliding stage. The initialization stage is only applicable to the first received block of \( N_c \) OFDM symbols (i.e., \( n \leq N_c \)), whereas the sliding stage applies to each of the following OFDM symbols (i.e., \( n > N_c \)) while making use of the \( (N_c - 1) \) previously estimated (using reduced ICI) time-averaged complex gains. The initialization and sliding stages proceed as follows.

**Initialization:**

\[ i \leftarrow 1 \]

if (initialization stage);

\[
y_{p(i)} = [y_{p(1,i)}, \ldots, y_{p(N_c,i)}] \]

where \( y_{p(1,i)} = y_{p(n)} \) \( n = 1, \ldots, N_c \)

**else** (sliding stage);

\[ n \leftarrow n + 1 \]

\[
\{A_{LS}[k,m], k = 1, \ldots, N_c - 1\} = \{A_{LS}[k,m], k = 2, \ldots, N_c\}
\]

\[ y_{p(n,i)} = y_{p(n)} \]
recursion:
1) if (initialization stage): \( A_{LS}^{T} = G Y_p(i) \)
   \( \) else (sliding stage): \( a_{LS} = G Y_p(n) \)
   \( \{ |A_{LS}|_{N_c,m}, m = 1, \ldots, L \} = \{ |a_{LS}|_{m}, m = 1, \ldots, L \} \)
2) \( C_{des} = T^{-1} A_{LS} \)
3) compute the channel matrix using (13)
   \( \) if (initialization stage): \( H_{(n,i)}, n = 1, \ldots, N_c \)
   \( \) else (sliding stage): \( H_{(N_c,i)} \)
4) remove the pilot ICI from the received data subcarriers \( y_{d(n)} \)
5) detection of data symbols \( x_{d(n,i)} \)
6) \( y_{p(n,i+1)} = y_{p(n)} - H_{p(n,i)} \hat{x}(n,i) \)
7) \( t \leftarrow t + 1 \)

where \( i \) represents the iteration number. Notice that at the end of the initialization stage, \( n = N_c \).

D. Computational Complexity

The purpose of this section is to determine the implementation complexity in terms of the number of multiplications needed for the sliding stage. The matrices \( F, F_p, G, T^{-1}, \) and \( M_{(n,d)} \) are precomputed and stored if the pilot subcarriers are fixed and the delays are invariant for a great number of OFDM symbols. The complexity of the LS estimator of \( a \) in step 1 is \( L \times N_p \), and for the estimation of \( N_c \) polynomial coefficients in step 2, it is \( L \times N_c^2 \). The computational cost of computing the channel matrix \( H_{(t)} \) in step 3 is \( N_c(N + L) \), which is less than that in [2], which is \( LN^2(N + 1) \). The complexity of removing the ICI in steps 4–6 is \( N_p(N - N_p) + ((N - N_p)(N - N_p + 1))/2 \) + \( N_p(N - 1) \). In conclusion, the significant reduction in computational complexity, in comparison with that found in [2], is mainly due to the fact that the calculation of the channel matrix is based on polynomial coefficients with no need to construct complex gain time variations using low-pass interpolation (LPI).

E. MSE Analysis

The MSE between the \( l \)th exact complex gain and the \( l \)th estimated polynomial (characterized by \( N_c \) coefficients and fitted to the time average values within \( N_c \) OFDM symbols) is defined by

\[
MSE_l = \frac{1}{v N_c} E \left[ (\alpha_l - \hat{\alpha}_{des})^H (\alpha_l - \hat{\alpha}_{des}) \right] \quad (22)
\]

where \( \hat{\alpha}_{des} = V \hat{\alpha}_{LS} \) is the \( l \)th estimated polynomial, which gives (see Appendix B)

\[
MSE_l = MSE_{des} + \frac{1}{v N_c} g_l^H (R_1 + R_2) g_l - \frac{2}{v N_c} Re(r_3^T g_l) \quad (23)
\]

where \( g_l \) is the \( l \)th row of the matrix \( G \), and \( R_1, R_2, \) and \( r_3 \) are computed in Appendix B. The first component on the right-hand side is the MSE of the polynomial approximation, the second component is the MSE of the \( l \)th estimated polynomial, and the third component is the cross-covariance term. It should be noted that if the ICI are completely eliminated, then \( R_2 \) and \( r_3 \) are, respectively, a matrix/vector of zeros. Equation (23) thus becomes

\[
MSE_l (\text{without ICI}) = MSE_{des} + \frac{1}{v N_c} g_l^H R_1 g_l \quad (24)
\]

where the second component on the right-hand side is the MSE of the \( l \)th estimated polynomial without ICI. This component is due to the error in the estimator of \( a \) without ICI (see (33) in Appendix B), which in our algorithm is the error of the LS estimator without ICI (see (34) in Appendix B). The lower bound (LB) of the estimator of \( a \) (without ICI) thus leads to the LB of the MSE between the exact complex gain and the estimated polynomial MSE with known ICI is given by (see Appendix A)

\[
SCRB_a = \frac{1}{SNR} (F_p^H \text{diag}(x_p))^H \text{diag}(x_p) F_p \quad (25)
\]

where \( SNR = 1/\sigma^2 \) is the normalized signal-to-noise ratio (SNR). Hence, from (33) in Appendix B, the LB of the MSE between the \( l \)th exact complex gain and the \( l \)th estimated polynomial is given by

\[
LB_l = MSE_{des} + G \times |SCRB_a|_{1,1} \quad (26)
\]

where \( G = ||V||^2/v N_c \) is a noise amplification gain. Interpreting the right-hand side of (26), the first component is the model error MSE_{des} that depends on \( f_d T \) and \( N_c \), whereas the second component is the LB of the MSE of the \( l \)th estimated polynomial that depends on SNR and \( N_c \). Consequently, the number of coefficients \( N_c \) needs to be chosen such that an acceptable tradeoff can be found between model error and noise reduction. It can easily be shown that

\[
\begin{align*}
\{ & MSE_l (\text{with ICI}) > LB_l \} \\
\{ & MSE_l (\text{without ICI}) = LB_l \}
\end{align*}
\quad (27)
\]

V. Simulation Results

In this section, the theory described earlier is demonstrated by simulation, and the performance of the iterative algorithm is tested. The MSE and the bit error rate (BER) performances are examined in terms of the average SNR [8], [9] and the maximum Doppler spread \( f_d T \) (normalized by \( 1/T \)) for the Rayleigh channel. The normalized channel model is Rayleigh as recommended by GSM Recommendations 05.05 [12], [13] using the parameters shown in Table I. A 4QAM-OFDM system is used with normalized symbols:
Fig. 3. Comparison of MSE for $N_c = 2$ and $3$ at SNR = 20 and 40 dB.

$N = 128$ subcarriers, $N_g = N/8$ subcarriers, $N_p = 16$ pilots (i.e., $L_f = 8$), and $1/T_s = 2$ MHz. The BER performance is evaluated under a relatively rapid time-varying channel using the values $f_dT = 0.05$ and $f_dT = 0.1$, which correspond to a vehicle driven at speeds $V_m = 140$ km/h and $V_m = 280$ km/h (high-speed train), respectively, for $f_c = 5$ GHz.

Table I

<table>
<thead>
<tr>
<th>Path Number</th>
<th>Average Power (dB)</th>
<th>Normalized Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.219</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-4.219</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>-6.219</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-10.219</td>
<td>3.2</td>
</tr>
<tr>
<td>5</td>
<td>-12.219</td>
<td>4.6</td>
</tr>
<tr>
<td>6</td>
<td>-14.219</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 5 provides a comparison between the MSE of the exact complex gain and the estimated polynomial in terms of $f_dT$ for $N_c = 2$ and $3$ at SNR = 20 and 40 dB. It is observed that for moderate values of SNR, the approximation achieved with $N_c = 2$ coefficients is better than that found using $N_c = 3$ coefficients. However, for high values of SNR, the opposite tendency is observed. This is due to the noise component in (23) and the third coefficient that is poorly estimated, particularly in the case of low SNR, because it is negligible compared to the noise level [1]. However, this difference between the MSE does not have a strong influence on the BER, as shown in Fig. 4.

Fig. 5 illustrates the evolution of MSE as the number of iterations progresses, as a function of SNR, for $f_dT = 0.1$. It is found that, with all ICIs, the MSE obtained by simulation agrees with the theoretical value given in (23). After only one iteration, a great improvement is realized, and the MSE is very close to the LB of our algorithm, particularly in regions of low and moderate SNR. This is because at low SNR, the noise is dominant with respect to the ICI level, whereas for high SNR, the ICI is not completely removed due to data symbol detection errors. Fig. 5 also shows that, for $f_dT = 0.1$ and SNR $\leq 30$ dB, the MSE of the polynomial approximation $\text{MSE}_{\text{des}}$ is negligible, and the main contribution to the MSE is that produced by the LS estimator. In this case, from (26), we indeed have $\text{LB}_1 \approx \mathcal{G} \times [\text{SCR}_{\mathbf{R}_1}]_{t,t}$ since $\text{MSE}_{\text{des}}$ is negligible when compared to SCR, as shown by comparing Fig. 1 with Fig. 11. To find the smallest possible LB, we thus have to choose $N_c = 2$ since $\mathcal{G}$ increases as a function of $N_c$, as shown in Table II. However, for high SNR levels, LB asymptotically tends toward $\text{MSE}_{\text{des}}$, which means that the smallest possible LB will be achieved when $N_c > 2$.

Fig. 6 shows the BER performance of our proposed iterative algorithm for $N_c = 2$ when compared with that achieved using conventional methods (LS and LMMSE criteria with LPI in the frequency domain) [6], [8], our previously proposed algorithm [2], and the SIS algorithm with perfect channel knowledge for $f_dT = 0.05$ and $f_dT = 0.1$. As a reference, we also plot the performance obtained with perfect channel and ICI knowledge.

Table II

<table>
<thead>
<tr>
<th>$N_c$</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{G}$</td>
<td>1.17</td>
<td>1.39</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Fig. 4. Comparison of BER for $N_c = 2$ and $3$ at SNR = 20 and 40 dB.

Fig. 5. MSE of the polynomial approximation for $f_dT = 0.1$ and $N_c = 2$.
This result shows that our algorithm has better performance than conventional methods and our previously published algorithm [2]. Moreover, the approach presented here enables an improvement in BER to be achieved after each iterative step, because each iteration necessarily results in an improvement in the estimation of ICI. After two iterations, a significant improvement occurs; the performance of our algorithm comes very close to that found with the SIS algorithm using perfect channel knowledge. For high values of SNR, our algorithm does not achieve the same performance as with perfect channel and ICI knowledge because an error floor remains due to data symbol detection error. This error floor could be decreased by using a detection scheme that is better than the SIS scheme.

Fig. 7 shows the BER in terms of $N_p$ for $f_dT = 0.1$, $N_c = 2$, and SNR = 20 dB. It is obvious that when the number of pilots is increased, the performance will improve. It is interesting to note that the results presented here demonstrate that with a lesser number of pilots, our algorithm has better performance than conventional methods.

Fig. 8 shows the BER performance of our proposed iterative algorithm for $N_c = 2$ and $f_dT = 0.1$ with IEEE 802.11a standard channel coding [21]. The convolutional encoder has a rate of 1/2, its polynomials are $P_0 = 133_8$ and $P_1 = 171_8$, and the interleaver is a bit-wise block interleaver with 16 rows and 14 columns. It can clearly be seen that a significant improvement in BER occurs with channel coding, and that for high SNR, there is always an error floor due to data symbol detection errors.

Fig. 9 shows the BER performance after three iterations of our proposed iterative algorithm for $N_c = 2$ and $f_dT = 0.1$ with imperfect delay knowledge. SD denotes the standard deviation of the time delay errors (modeled as zero mean Gaussian variables). It can be noticed that the algorithm is not very sensitive to a delay error of $SD < 0.1T_s$. By using the ESPRIT method [9] to estimate the delays, we have $SD < 0.05T_s$ for all SNRs, as shown in Fig. 10. When combined with the ESPRIT method, our algorithm thus has negligible sensitivity to delay errors.
Fig. 9. Comparison of BER for the case of imperfect knowledge of delays for $N_c = 2$ and $f_d T = 0.1$.

![Graph showing BER comparison for different delay estimation scenarios](image)

Fig. 10. Delay estimation errors for the fourth and sixth paths using the ESPRIT method [9] (estimated correlation matrix averaged over 1000 OFDM symbols, i.e., 0.072 s) for $f_d T = 0.1$.

![Graph showing delay estimation errors for different SNRs](image)

Appendix A

CRB for the Estimator of $\alpha$

In this Appendix, we calculate the CRB for the estimation of $\alpha$ based on the received pilot subcarriers $y_p$ of the current OFDM symbol. If it is assumed that $\text{ICL}_p = \mathbf{H}_p x$ in (16) is known, the vector $y_p$ for a given $\alpha$ is a complex Gaussian with mean vector $\mathbf{m} = \text{diag}(\{p\}) \mathbf{F}_p \alpha + \text{ICL}_p$ and covariance matrix $\Omega_1 = \sigma^2 T N_p$. Thus, the probability density function $p(y_p | \alpha)$ is defined as

$$p(y_p | \alpha) = \frac{1}{|\pi \Omega_1|} e^{-(y_p - \mathbf{m})^H \Omega_1^{-1}(y_p - \mathbf{m})}.$$ 

Since $\alpha$ is a complex Gaussian vector with zero mean and covariance matrix $\Omega_2$, the probability density function of $\alpha$ can be defined as

$$p(\alpha) = \frac{1}{|\pi \Omega_2|} e^{-\alpha^H \Omega_2^{-1} \alpha}$$

where $\Omega_2$ is a $L \times L$ diagonal matrix of elements given by

$$[\Omega_2]_{l,l} = E[|\{a\}_l|^2] = \frac{\sigma^2}{N} \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} J_0(2\pi f_d T_s(q_1 - q_2)).$$

The SCRB and the Bayesian CRB (BCRB) for the estimator of $\alpha$ are defined as [14]

$$\text{SCRB}_\alpha = \left(-E \left[ \frac{\partial^2}{\partial \alpha^* \partial \alpha} \ln p(y_p | \alpha) \right] \right)^{-1}$$

$$\text{BCRB}_\alpha = \left(-E \left[ \frac{\partial^2}{\partial \alpha^* \partial \alpha} \ln p(y_p, \alpha) \right] \right)^{-1}$$

(28)

where $p(y_p, \alpha) = p(y_p | \alpha) p(\alpha)$ is the joint probability density function of $y_p$ and $\alpha$, and the expectation is computed over $y_p$ and $\alpha$. Notice that SCRB and BCRB are used for the estimation of deterministic and random variables, respectively.

The results of the second derivatives of $\ln(p(y_p | \alpha))$ and $\ln(p(y_p, \alpha))$ with respect to $\alpha$ are given by

$$\frac{\partial^2}{\partial \alpha^* \partial \alpha^T} \ln (p(y_p | \alpha)) = -\mathbf{F}_p^H \text{diag}(\{p\}) H \Omega_1^{-1} \text{diag}(\{p\}) \mathbf{F}_p$$

$$\frac{\partial^2}{\partial \alpha^* \partial \alpha^T} \ln (p(y_p, \alpha)) = -\mathbf{F}_p^H \text{diag}(\{p\}) H \Omega_1^{-1} \text{diag}(\{p\}) \mathbf{F}_p - \Omega_2^{-1}.$$ 

(29)

Hence, substituting (29) into (28) yields

$$\text{SCRB}_\alpha = \sigma^2 \left( \mathbf{F}_p^H \text{diag}(\{p\}) H \text{diag}(\{p\}) \mathbf{F}_p \right)^{-1}$$

$$\text{BCRB}_\alpha = \left( \frac{1}{\sigma^2} \mathbf{F}_p^H \text{diag}(\{p\}) H \text{diag}(\{p\}) \mathbf{F}_p + \Omega_2^{-1} \right)^{-1}.$$ 

It should be noticed that in our specific problem, SCRB is independent of $\alpha$. SCRB thus defines the LB if the a priori distribution of $\alpha$ is not used in the estimation method, whereas BCRB takes this information into account. This is illustrated in Fig. 11, which plots $\text{SCRB} = \text{Tr}(\text{SCRB}_\alpha)$ and $\text{BCRB} = \text{Tr}(\text{BCRB}_\alpha)$ as a function of SNR for the channel defined.
in Table I with $N = 128$, $N_p = 16$, and $f_d T = 0.1$. It can be observed that there is a small difference between SCRB and BCRB only at low values of SNR. We can thus compare the MSE of our LS estimator of $\alpha$ with SCRB instead of BCRB. Moreover, for a known ICI, the optimal estimators of deterministic $\alpha$ and random (Gaussian) $\alpha$ are the LS and maximum likelihood (ML) estimators, respectively. The LS estimator was used (for deterministic $\alpha$) because it requires less information than the ML estimator.

**APPENDIX B**

**MSE of the Complex Gains Estimator**

Let $\Delta_p = [\text{ICI}_{p(n-Nc+1)}, \ldots, \text{ICI}_{p(n)}]$ with $\text{ICI}_{p(n)} = H_{p(n)} x_{p(n)}$ and $W_{p} = w_{p(n-Nc+1)}, \ldots, w_{p(n)}$. The error matrix of the LS estimator over $N_c$ OFDM symbols is given by

$$\mathcal{E} = \mathbf{A}_{LS}^T - \mathbf{G}^T = \mathbf{G}(\Delta_p + W_{p}).$$

The error between the $l$th exact complex gain and the $l$th estimated polynomial is given by

$$e_l = \mathbf{x}_l - \mathbf{V} \mathbf{x}_{LS} = \mathbf{e}_{des} - \mathbf{V} e_l$$

(30)

$$e_l = \mathbf{e}_{des} - \mathbf{V}(\Delta_p + W_p) g_l$$

(31)

where $e_l^T$ and $g_l^T$ are the $l$th rows of the matrices $\mathcal{E}$ and $\mathbf{G}$, respectively. Since the noise and the ICI are uncorrelated, the MSE between the $l$th exact complex gain and the $l$th estimated polynomial is given by (23), where $R_1$, $R_2$, and $r_3^T$ are defined by

$$R_1 = E\left[\mathbf{W}_{p}^H \mathbf{V} \mathbf{W}_{p}\right] = \sigma^2 ||\mathbf{V}||^2 \mathbf{I}_{N_p}$$

$$R_2 = E\left[\mathbf{\Delta}_{p}^H \mathbf{V} \mathbf{\Delta}_{p}\right]$$

$$r_3^T = E\left[\mathbf{e}_{des}^H \mathbf{V} \mathbf{\Delta}_{p}\right].$$

ICI_{p(n)} can be written as the sum of two components

$$\text{ICI}_{p(n)} = \text{ICI}_{pp(n)} + \text{ICI}_{dd(n)}$$

where $\text{ICI}_{pp(n)} = H_{pp(n)} x_{p(n)}$, and $\text{ICI}_{dd(n)} = H_{dd(n)} x_{d(n)}$, in which $H_{pp(n)}$ and $H_{dd(n)}$ are $N_p \times N_p$ and $N_p \times (N - N_p)$ matrices, respectively, whose elements are given by

$${\mathbf{H}_{pp(n)}}_{k,m} = \begin{cases} \left(\mathbf{H}_{n}\right)_{p_k,p_m}, & \text{if } k \neq m \\ 0, & \text{if } k = m \end{cases}$$

where $\{p_k\}$’s are defined in (15), and $m \in \left[1, N - P \right]$. Hence, the matrix $R_2$ becomes $R_2 = R_{pp} + R_{dd}$, where $R_{pp} = E[\Delta_{pp} \mathbf{V}^H \mathbf{V} \Delta_{pp}^T]$, and $R_{dd} = E[\Delta_{dd}^* \mathbf{V}^H \mathbf{V} \Delta_{dd}^T]$, since the data symbols and coefficients $\{\mathbf{H}_{n}\}_{k,m}$ are uncorrelated. The data symbols are normalized (i.e., $E[x_{(u)}|d_1|x_{(u)}^*|d_2]$) such that the elements $[R_{pp}]_{k,m}$, $[R_{dd}]_{k,m}$, and $[r_3]_k$ with $k, m \in [1, N_p]$ can be calculated as

$$[R_{pp}]_{k,m} = \sum_{u=1}^{N_c} \sum_{u_1=1}^{N_c} \sum_{u_2=1}^{N_c} |V_{u,u_1}|^2 |V_{u,u_2}|^2 \mathbf{Z}_{p(k,m)}_{u_1,u_2}$$

$$[R_{dd}]_{k,m} = \sum_{u=1}^{N_c} \sum_{u_1=1}^{N_c} \sum_{u_2=1}^{N_c} |V_{u,u_1}|^2 |V_{u,u_2}|^2 \mathbf{Z}_{d(k,m)}_{u_1,u_2}$$

$$[r_3]_k = E\left[\sum_{u=1}^{N_c} \sum_{u_1=1}^{N_c} \sum_{u_2=1}^{N_c} |V_{u,u_1}|^2 |V_{u,u_2}|^2 \mathbf{Z}_{3(k)}_{u_1,u_2}\right]$$

where $\{\mathbf{Z}_{p(k,m)}_{u_1,u_2}\}$, $\{\mathbf{Z}_{d(k,m)}_{u_1,u_2}\}$, $\{\mathbf{Z}_{1(k)}_{u,u_1}\}$, and $\{\mathbf{Z}_{2(k)}_{u,u_1}\}$ are given by (32), shown at the top of the next page. Notice that the elements of the matrix $R_2$ and $r_3$ depend on known pilot symbols.

If the ICI are completely eliminated, then the elements of $\mathcal{E}$ are uncorrelated with respect to each other and the elements of $\mathbf{e}_{des}$. Thus, from (30), we can write

$$\text{MSE}_{l} \text{ (without ICI)} = \text{MSE}_{des} + \frac{||\mathbf{V}||^2}{v N_c} E\left[\mathbf{e}_{l}^H \mathbf{e}_{l}\right].$$

(33)

Combining (33) and (31) for the case of the LS estimator thus leads to

$$\text{MSE}_{l} \text{ (without ICI)} = \text{MSE}_{des} + \frac{||\mathbf{V}||^2 ||g_l||^2}{v N_c \text{SNR}}.$$  

(34)

**APPENDIX C**

**SIS METHOD**

The received data subcarriers, without contributions from pilot subcarriers, are given by

$$y_d = H_d x_d + w_d$$

where $x_d$ is the transmitted data, $y_d$ is the received data, $w_d$ is the noise at the data subcarrier positions given by $(N-N_p) \times 1$ vectors, and $H_d$ is a $(N-N_p) \times (N-N_p)$ data channel matrix obtained by eliminating rows and columns at the $P$ position in the channel matrix $H$.

Through the implementation of a SIS scheme, with optimal ordering and one-tap frequency equalizer, the data can be
\[
[Z_{p(k,m)}]_{u_1,u_2} = E \left[ (\Delta_{pp})_{m,u_1} \right] = E \left[ \sum_{d_1 = 1}^{P_{Ny}} \sum_{d_2 = 1}^{P_{Ny}} \sigma_{\alpha_1}^2 \left[ x_{(u_1)} \right]_{d_1} \left[ x_{(u_2)} \right]_{d_2} e^{-j2\pi \frac{d_1-d_2}{N}} \right] \\
= \frac{1}{N^2} \sum_{d_1 = 1}^{P_{Ny}} \sum_{d_2 = 1}^{P_{Ny}} \sigma_{\alpha_1}^2 \left[ x_{(u_1)} \right]_{d_1} \left[ x_{(u_2)} \right]_{d_2} J_0 \left( 2\pi f_d T_s \right) \left( (q_1 - q_2) + (u_1 - u_2) \right)
\]

\[
[Z_{d(k,m)}]_{u_1,u_2} = E \left[ (\Delta_{pd})_{m,u_1} \right] = E \left[ \sum_{d_1 = 1}^{P_{Ny}} \sum_{d_2 = 1}^{P_{Ny}} \sum_{q_1 = 0}^{N-1} \sum_{q_2 = 0}^{N-1} e^{j2\pi \frac{d_1-d_2}{N}} J_0 \left( 2\pi f_d T_s \right) \right] \\
= \frac{\delta_{u_1,u_2}}{N^2} \sum_{l=1}^{L} \sum_{d_1 = 1}^{P_{Ny}} \sum_{d_2 = 1}^{P_{Ny}} \sum_{q_1 = 0}^{N-1} \sum_{q_2 = 0}^{N-1} e^{j2\pi \frac{d_1-d_2}{N}} J_0 \left( 2\pi f_d T_s \right) \left( (q_1 - q_2) + (u_1 - u_2) \right)
\]

\[
[Z_{1(k)}]_{u_1,u_1} = E \left[ \sigma_{\alpha_1}^2 \left( (u-1) T_s \right) \left( \Delta_{pp} \right) \right] = E \left[ \sum_{d_1 = 1}^{P_{Ny}} \sum_{d_2 = 1}^{P_{Ny}} \sum_{q_1 = 0}^{N-1} \sum_{q_2 = 0}^{N-1} e^{j2\pi \frac{d_1-d_2}{N}} J_0 \left( 2\pi f_d T_s \right) \left( (q_1 - q_2) + (u_1 - u_2) \right)
\]

\[
[Z_{2(k)}]_{u_1,u_2} = E \left[ \sigma_{\alpha_1}^2 \left( \Delta_{pp} \right) \right] = E \left[ \sum_{d_1 = 1}^{P_{Ny}} \sum_{d_2 = 1}^{P_{Ny}} \sum_{q_1 = 0}^{N-1} \sum_{q_2 = 0}^{N-1} e^{j2\pi \frac{d_1-d_2}{N}} J_0 \left( 2\pi f_d T_s \right) \left( (q_1 - q_2) + (u_1 - u_2) \right)
\]

estimated. Optimal ordering of the data channel matrix \( H_d \), which is computed from the largest to the smallest magnitude of the diagonal elements, is given by

\[
O = \{ O_1, O_2, \ldots, O_{N-N_p} \}
\]

\[
i < j \text{ if } \| H_d |_{O_i, O_j} \| > \| H_d |_{O_j, O_j} \|
\]

The detection algorithm can now be described as follows:

\textbf{initialization:}

\[
i \leftarrow 1
\]

\[
O = \{ O_1, O_2, \ldots, O_{N-N_p} \}
\]

\[
Y_{d(i)} = Y_d
\]

\textbf{recursion:}

\[
\begin{align*}
[X_{ed}]_{O_i} &= [Y_{d(i)}]_{O_i} / |H_d|_{O_i, O_i} \\
[X_d]_{O_i} &= Q(\{X_{ed}\}_{O_i}) \\
Y_{d(i+1)} &= Y_{d(i)} - [X_d]_{O_i} h_d_{O_i}
\end{align*}
\]

\[
i \leftarrow i + 1
\]

where \( Q(.) \) denotes the quantization operation appropriate to the constellation in use, and \( h_d_{O_i} \) is the \( O_i \)th column of the data channel matrix \( H_d \). Notice that the complexity could be reduced, with very little loss in performance, if SIS were processed on a small number of adjacent subcarriers only [20].

\textbf{REFERENCES}


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